

Spacecraft Design Loads

6.1 Introduction

Launch vehicle and spacecraft low frequency loads are driven by transients such as engine ignition, engine shutdowns, wind gusts or wind shears, and quasi-static loads. Other environments are acoustics, random vibration, sine vibration, and shock.

These environments are driven by the ascent profile, which includes the events listed in Table 6.1.

Table 6.1 Sources of launch vehicle environments [Yunis 2005]

	Acoustics	Random Vibration	Sine Vibration	Shock
Lift-off	X	X		
Aerodynamics /Buffet	X	X		
Separation (stage, fairing, spacecraft)				X
Motor burn /Combustion/ POGO		X	X	

The maximum loads (flight limit loads) at any stage in the life cycle of a spacecraft or other space system are used to design and dimension the primary, secondary and other parts.

The dynamic mechanical loads that occur during the lifetime of a spacecraft are:

- Handling loads
- Transportation loads
- Vibration tests required for the qualification of the spacecraft structure
 - Sinusoidal vibrations
 - Random vibrations
 - Acoustic pressures

- Dynamic loads during launch
 - Steady-state acceleration (inertia loads)
 - Sinusoidal vibrations
 - Random vibrations
 - Acoustic loads
 - Shock loads
 - Pressure variations
- Re-entry loads
- (Emergency) landing loads (STS)
- Loads following launch
 - Transfer orbit loads
- Loads/influences on the spacecraft in orbit (In-service loads)
 - Extension of folded elements, such as solar panels, antennas, etc.
 - Temperature gradients
 - 0g loads
 - Micro-meteorites / Debris

The dynamic loads during launch of the spacecraft are generally the highest for the basic structure. The test loads are dealt with in a later chapter.

Foldable structures experience different loads during launch than in orbit around the earth.

In the following sections launch loads and micro-meteoroid / debris will be covered, and specifically:

- Steady-state static loads as a result of:
 - The propulsion of the engine
 - Crosswind loads
 - Manoeuvres
- Mechanical dynamic loads that are a result of unsteady combustion of the engine(s), the turbulent flows along the rocket and the noise of the exhaust (especially during the initial phase of launch). These enforced mechanical vibrations (base excitation) transferred via the interface of the spacecraft with the Launch Vehicle, are in general:
 - Sinusoidal vibrations
 - Random vibrations
 - Shock loads
- Acoustic loads (sound pressures) as a result of exhaust noises and the turbulent flows along the launch vehicle.
- Shock loads as a result of the separation of the stages and the separation of the spacecraft from the launch vehicle, the ignition and the stopping of the engines. The separation of the spacecraft results in the highest shock load.
- Pressure changes. The absolute pressure decreases during launch, which can influence the systems unless suitable ventilation systems have been fitted.
- Micro-meteorites/Debris. Parts, boxes and instruments mounted on the outside of the spacecraft are exposed to micro meteorites and man-made debris.

6.2 Transportation load factors

The typical transportation and handling load factors are given in Table 6.2.

Table 6.2 Transportation limit load factors [NASA-HDBK-7005]

Medium/Mode	Longitudinal load factors	Lateral load factors	Vertical load factors
Water	± 0.5	± 2.5	± 2.5
Air	± 3.0	± 1.5	± 3.0
Ground			
• Truck	± 3.5	± 2.0	± 6.0
• Rail (humping shocks)	± 6.0 to ± 30.0	± 2.0 to ± 5.0	± 4.0 to ± 15.0
• Rail (rolling)	± 0.25 to ± 3.0	± 0.25 to ± 0.75	± 0.2 to ± 3.0
• Slowly moving dolly	± 3.1	± 0.75	± 2.0

The transportation loads should be included in the design analysis unless special protection is provided to assure that they contribute negligible damage compared with the other (flight) loads.

6.3 Steady-State Loads

The maximum steady-state acceleration in the launch direction occur at the end of the propulsion phase of a rocket stage. The acceleration increases because the mass of the launch vehicle decreases, while the overall thrust remains the same. An example of the acceleration is illustrated in Fig. 6.1.

The vibrations are superimposed on the steady state acceleration.

The lateral steady-state accelerations are usually much smaller than the acceleration in the launch direction.

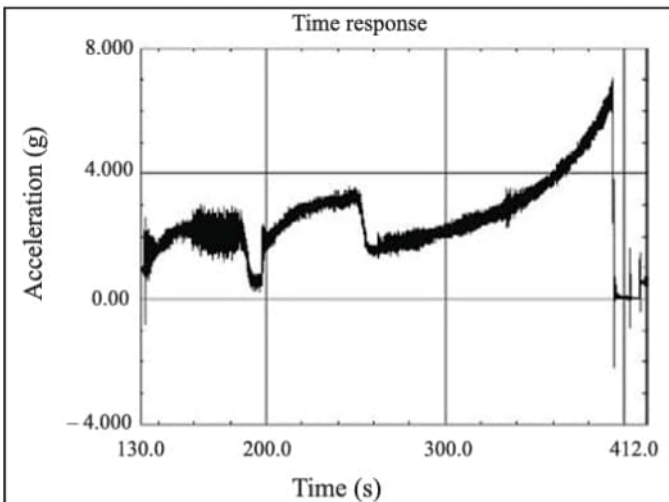


Fig. 6.1 Acceleration versus time Delta 2 launch vehicle (courtesy FEMCI, NASA GSFC)

The maximum steady state accelerations of various launch vehicles are shown in Table 6.3.

Table 6.3 Maximum steady-state acceleration

Launch vehicle	Maximum steady-state acceleration	
	Longitudinal [g]	Lateral [g]
Ariane 4	4.5	0.2
Ariane 5	4.25	0.2
Atlas	5.5	0.4
Delta 2	5.5–7 ^a	0
Pegasus	7–10	0
Proton	4	0
Long March 2E	5.2	0.6
Long March 3	5.5	0.6

a. Depends on mass of spacecraft

6.4 Mechanical Dynamic loads

The mechanical dynamic loads during launch can be subdivided into:

- Low frequency sinusoidal vibrations in a frequency domain of 5–100 Hz.
- Random vibrations in a frequency range of 20 – 2000 Hz.

An example of the low frequency acceleration is illustrated in Fig. 6.2 and an example of high frequency acceleration is shown in Fig. 6.3.

6.4.1 Sinusoidal loads

Low frequency sinusoidal vibrations occur as a result of the interaction between launch vehicle mode forms and loads occurring during [NASA Practice No. PT-TE-1406, Lalanne 2002a]:

- Lift-off, the fast build-up of thrust causes a shock load that excites the low frequency domain.
- Combustion of the engines, during combustion of the engines sinusoidal vibrations occur, both in, and adjacent to, the launch direction.

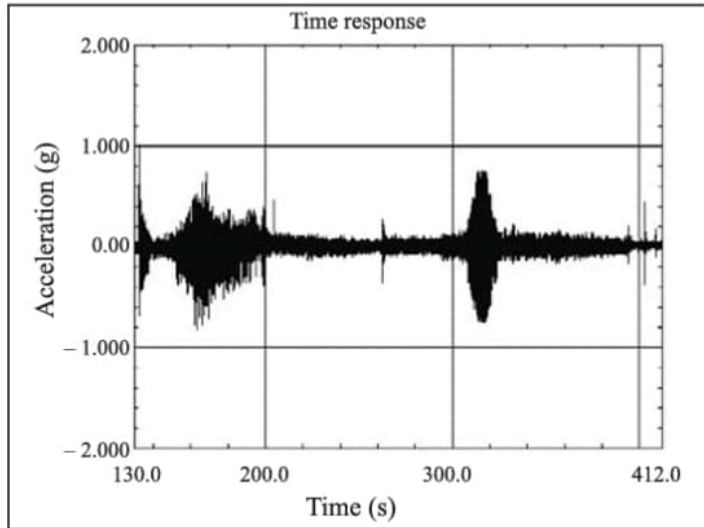


Fig. 6.2 Low frequency acceleration versus time (courtesy FEMCI, NASA GSFC)

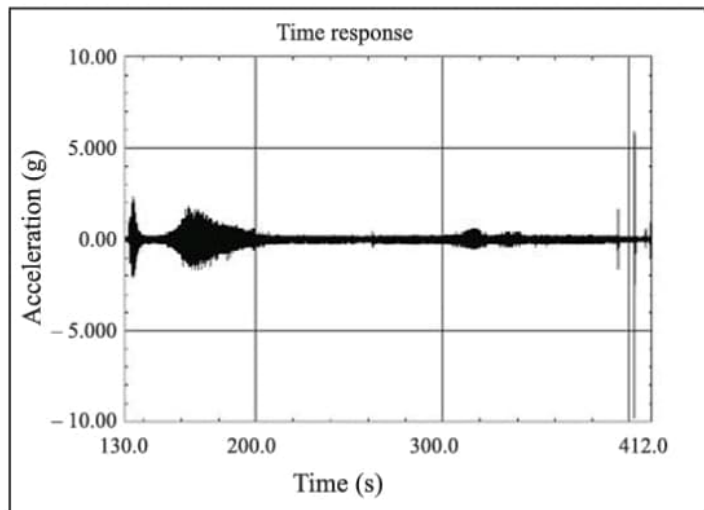


Fig. 6.3 High frequency acceleration versus time (courtesy FEMCI, NASA GSFC)

- POGO (a stick with a spring on the bottom). Even though engineers will go to great lengths to reduce the effects of POGO vibrations, they are still observed just before the burn up of a stage.

The maximum sinusoidal vibrations for a DELTA 7925 L/V are summarized in Table 6.4:

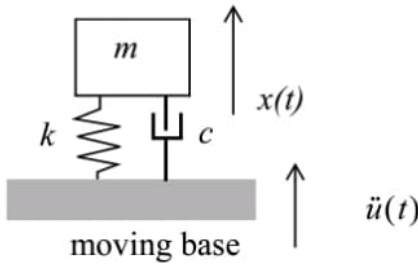
Table 6.4 Sinusoidal vibrations

	Frequency (Hz)	Acceleration (g)
Launch direction	5 – 6.2	12.7 mm double amplitude
	6.2 – 100	1.0
Lateral direction	5 – 100	0.7

Enforced accelerations

A SDOF system with a discrete mass m , a damper element c and a spring element k , is placed on a moving base which is accelerated with an acceleration $\ddot{u}(t)$. The resulting displacement of the mass is $x(t)$. A relative motion $z(t)$ is introduced which is the displacement of the mass with respect to the base. This is defined as:

$$z(t) = x(t) - u(t). \quad (6.1)$$

**Fig. 6.4** Enforced acceleration

The equation of motion for $z(t)$ can be written as

$$\ddot{z}(t) + 2\zeta\omega_n\dot{z}(t) + \omega_n^2z(t) = -\ddot{u}(t). \quad (6.2)$$

The enforced acceleration of the SDOF system is transformed into an external force. The absolute displacement $x(t)$ can be calculated with (6.1) or

$$\ddot{x}(t) = \ddot{z}(t) + \ddot{u}(t) = -2\zeta\omega_n\dot{z}(t) - \omega_n^2z(t). \quad (6.3)$$

Using the initial conditions, the displacement $z(0)$ and the velocity $\dot{z}(0)$, the solution of (6.2) for $z(t)$ is

$$z(t) = z(0)e^{-\zeta\omega_n t} \left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d t \right) + \dot{z}(0)e^{-\zeta\omega_n t} \frac{\sin\omega_d t}{\omega_d} - \int_0^t e^{-\zeta\omega_n \tau} \frac{\sin\omega_d \tau}{\omega_d} \ddot{u}(t-\tau) d\tau. \quad (6.4)$$

The first part of (6.4) is dependent on the initial conditions and will damp out very quickly. Therefore we will focus on the particular (or steady-state) solution.

Generally, the harmonic vibration is expressed in complex numbers ($j = \sqrt{-1}$)

$$z(t) = Z(\omega)e^{j\omega t}, \quad (6.5)$$

with the following definition of the Fourier transform pair

$$Z(\omega) = \int_{-\infty}^{\infty} z(t)e^{-j\omega t} dt, \quad (6.6)$$

and

$$z(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z(\omega)e^{j\omega t} d\omega. \quad (6.7)$$

The velocity becomes

$$\dot{z}(t) = j\omega Z(\omega)e^{j\omega t} = \dot{Z}(\omega)e^{j\omega t}, \quad (6.8)$$

and the acceleration will be

$$\ddot{z}(t) = (j\omega)^2 Z(\omega)e^{j\omega t} = -\omega^2 Z(\omega)e^{j\omega t} = \ddot{Z}(\omega)e^{j\omega t}. \quad (6.9)$$

The multiplication with j will rotate the vector (e.g. $Z(\omega)$) 90° in the positive direction in the Argand diagram (Wessel's geometry, [Nahin 1998]).

The complex number $j = \sqrt{-1}$ is called the rotation operator.

Equation (6.2) becomes

$$(-\omega^2 + 2j\zeta\omega\omega_n + \omega_n^2)Z(\omega) = -\ddot{U}(\omega), \quad (6.10)$$

or

$$\ddot{Z}(\omega) = \frac{\omega^2 \ddot{U}(\omega)}{(-\omega^2 + 2j\zeta\omega\omega_n + \omega_n^2)} = H(\omega)(\ddot{U}(\omega)), \quad (6.11)$$

and

$$\ddot{X}(\omega) = \ddot{Z}(\omega) + \ddot{U}(\omega) = \{H(\omega) + 1\} \ddot{U}(\omega), \quad (6.12)$$

$$H_{\ddot{x}}(\omega) = H(\omega) + 1 = \frac{\omega^2}{\omega_n^2 \left(\left(1 - \frac{\omega^2}{\omega_n^2} \right) + 2j\zeta \frac{\omega}{\omega_n} \right)} + 1. \quad (6.13)$$

From this, three response regions can be determined:

1. $\frac{\omega}{\omega_n} < 1$ the transfer function $H_{\dot{x}}\left(\frac{\omega}{\omega_n}\right) \approx 1$, this is called stiffness controlled
2. $\frac{\omega}{\omega_n} = 1$ the transfer function $H_{\dot{x}}\left(\frac{\omega}{\omega_n}\right) = \frac{1}{2j\zeta} + 1 \approx \frac{1}{2j\zeta}$, this is called damping controlled
3. $\frac{\omega}{\omega_n} > 1$ the transfer function $H_{\dot{x}}\left(\frac{\omega}{\omega_n}\right) \approx 0$, this is called mass controlled

The transfer function $\left|H_{\dot{x}}\left(\frac{\omega}{\omega_n}\right)\right|$ is plotted in Fig. 6.5.

The sinusoidal or harmonic displacement $x(t)$ can be written as

$$x(t) = De^{j\omega t}, \quad (6.14)$$

where D is the amplitude of the sinusoidal displacement and ω is the excitation frequency (Rad/s). The radian frequency ω can be expressed as a number of cycles per second f (cps) or (Hz) with $\omega = 2\pi f$.

The velocity $\dot{x}(t)$ is the time derivative of the displacement

$$\dot{x}(t) = j\omega De^{j\omega t}. \quad (6.15)$$

It is observed that the velocity $\dot{x}(t)$ has a phase shift of $\frac{\pi}{2}$ radians with respect to the displacement $x(t)$.

The acceleration $\ddot{x}(t)$ is the time derivative of the displacement

$$\ddot{x}(t) = -\omega^2 De^{j\omega t}. \quad (6.16)$$

Similarly, it is also observed that the acceleration $\ddot{x}(t)$ has a phase shift of $\pm\pi$ radians with respect to the displacement $x(t)$.

At a frequency $f = 6.2$ Hz and an amplitude $D = \frac{0.0127}{2} = 0.00635$ m, the amplitude $\omega^2 D$ of the harmonic acceleration becomes
 $\omega^2 D = (2\pi \cdot 6.2)^2 \cdot 0.00635 = 9.64 \text{ m/s}^2 (\approx 1)g$

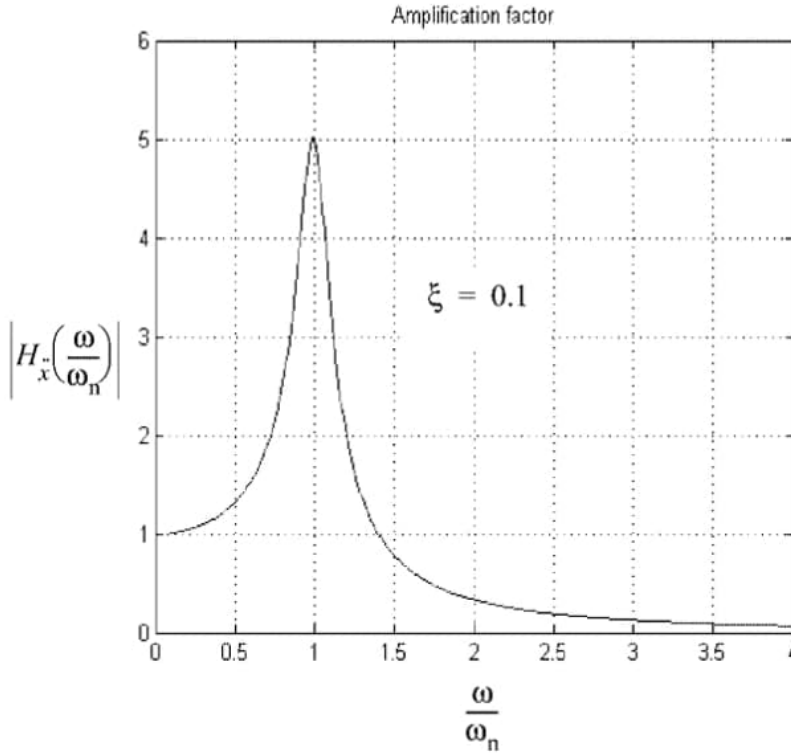


Fig. 6.5 Transfer function $\left| H_{\ddot{x}}\left(\frac{\omega}{\omega_n}\right) \right|$

Influence of the Natural Frequency

A two mass–spring system is illustrated in Fig. 6.6. The system with m_1 and k_1 represents a spacecraft, instrument or box, and the system with m_2 and k_2 represents the launch vehicle or spacecraft.

The application of the quasi-static loads is only allowed if the natural frequency of the combination of spacecraft & launch vehicle or the combination of spacecraft & instrument or box is well separated. In that case, the system with the highest natural frequency

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}}, \quad (6.17)$$

is significantly higher than the lowest natural frequency of the total system.

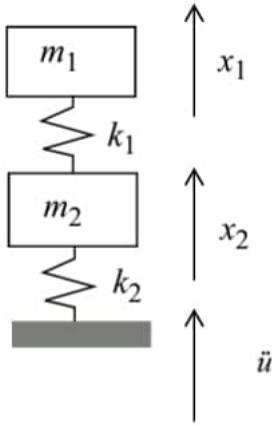


Fig. 6.6 Two mass-spring system

The lowest frequency of the total system can be estimated with Dunkerly's equation

$$\frac{1}{f_{\text{tot}}^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2}, \quad (6.18)$$

with $f_2 = \frac{1}{2\pi} \sqrt{\frac{k_2}{m_1 + m_2}}$. f_2 is the natural frequency of the total system with the system m_1 and k_1 acting as a rigid body. If f_1 and f_2 are far apart then lowest natural frequency of the total system becomes approximately $f_{\text{tot}} \approx f_2$. The motion is stiffness driven (see Fig. 6.5).

Another method to approximate the lowest natural frequency is with the Rayleigh Quotient

$$\omega^2 \approx R(q) = \frac{\{q\}^T [K] \{q\}}{\{q\}^T [M] \{q\}}, \quad (6.19)$$

where $[M]$ and $[K]$ are the mass and stiffness matrix respectively and $\{q\}$ is an assumed mode. We can take the static displacement under 1-g

$$\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = g \begin{Bmatrix} \frac{m_1}{k_1} + \frac{m_1 + m_2}{k_2} \\ \frac{m_1 + m_2}{k_2} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{f_1^2} + \frac{1}{f_2^2} \\ \frac{1}{f_2^2} \end{Bmatrix} = \begin{Bmatrix} \frac{f_2^2}{f_1^2} + 1 \\ 1 \end{Bmatrix} \quad (6.20)$$

If the ratio $\frac{f_2^2}{f_1^2} < 1$ then the ratio $\frac{q_1}{q_2} \approx 1$, the system m_1 and k_1 will act as a rigid

body with respect to the system m_2 and k_2 . In the case where $\frac{f_2^2}{f_1^2} \approx 1$ the system m_1 and k_1 will act as a mass–spring damper and will amplify considerably.

Example

A dynamic system is assumed to have $m_1 = 10$ kg, $m_2 = 150$ kg with $f_2 = \frac{1}{2\pi} \sqrt{\frac{k_2}{m_1 + m_2}} = 15$ Hz. It is also assumed that $f_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m_1}} = 40$ Hz.

The spring stiffness can then be calculated by $k_1 = (2\pi f_1)^2 m_1$ and $k_2 = (2\pi f_2)^2 (m_1 + m_2)$. The base excitation $\ddot{u} = 1$ m/s² over the frequency domain. The modal damping ratio is $\zeta = 0.02$. The transfer functions $|H_1(\omega)|$ and $|H_2(\omega)|$ will be calculated.

We see from Fig. 6.7 that the system m_1 and k_1 moves with about the same amplitude compared to the mass m_2 .

Assuming $f_1 = f_2 = 15$ Hz, the transfer functions $|H_1(\omega)|$ and $|H_2(\omega)|$ will be calculated again.

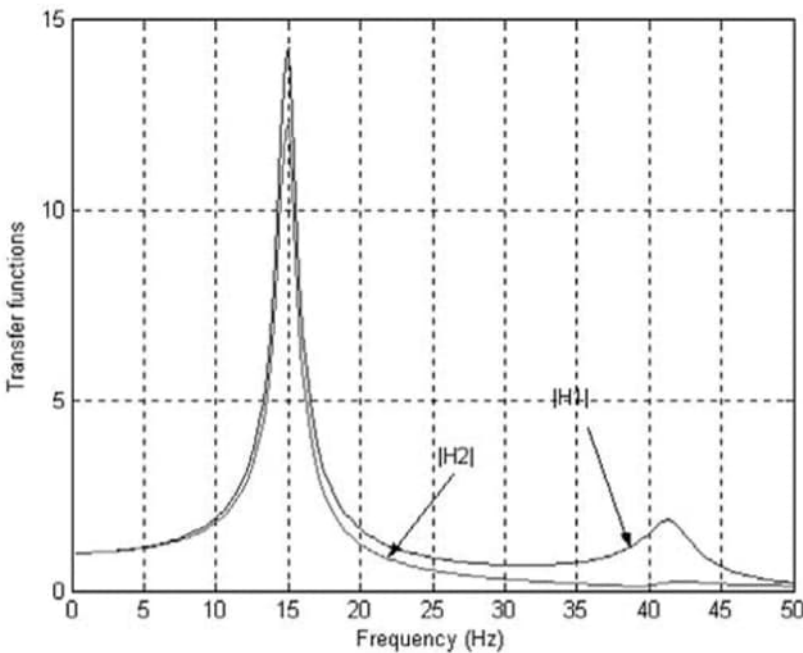


Fig. 6.7 Transfer functions

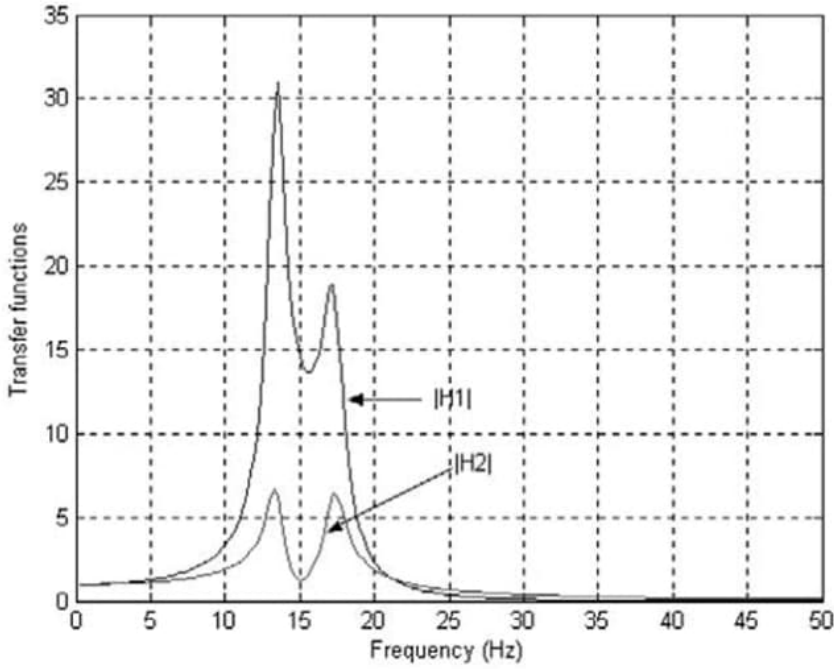


Fig. 6.8 Frequency transfer functions

We see from Fig. 6.8 that the system m_1 and k_1 moves with much higher amplitude compared with the mass m_2 . The system m_1 and k_1 acts as a tuned mass–spring system with the same natural frequency as the mass m_2 , however, the peak of mass m_2 is divided into two peaks with lower amplitude.

End of example

6.4.2 Random loads

Acoustic loads and boundary layer turbulence are transformed into mechanical vibrations in the launch vehicle, which affect the spacecraft at its base.

In the ARIANE 5 User’s Manual no random mechanical vibrations are specified. It is assumed that the acoustic loads will cover the random mechanical vibrations. There are many textbooks about random vibration, i.e. [Lalanne 2002c, Newland 1975].

In general random vibration loads are specified for instruments and equipment boxes, etc. Random mechanical loads are specified for the ARIANE 4 L/V (Table 6.5). These are valid at the base of the spacecraft.

It is assumed that the random accelerations are stationary and ergodic.

Table 6.5 Random vibrations

Frequency range (Hz)	Power Spectral Density (g ² /Hz)	rms acceleration (g)
20–150	+6dB/octave	7.3
150–700	0.04	
700–2000	–3dB/octave	

Power Spectral Density

The root mean square value (rms) value of a periodic signal $x(t)$ with a period

$T = \frac{1}{f}$ (s) is defined by:

$$x_{rms} = \left[\frac{1}{T} \int_{t_o}^{t_o+T} \{x(t)\}^2 dt \right]^{\frac{1}{2}}, \quad (6.21)$$

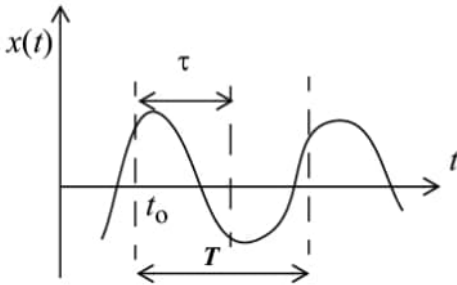
where t_o is an arbitrary starting time.

For a random signal $x(t)$ the x_{rms} is defined by

$$x_{rms} = \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt \right]^{\frac{1}{2}}. \quad (6.22)$$

The auto-correlation function of $x(t)$ is defined by

$$R_{xx}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)x(t+\tau) dt. \quad (6.23)$$

**Fig. 6.9** Periodic signal $x(t)$

The time shift τ is illustrated in Fig. 6.9

It can be observed from the (6.21) and (6.23) that

$$R_{xx}(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)x(t)dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = x_{\text{rms}}^2. \quad (6.24)$$

Example

A harmonic displacement is defined as $x(t) = D \sin(\omega t)$. Calculate the root mean square value of the displacement x_{rms} . The period time is $T = \frac{2\pi}{\omega}$, and is illustrated in Fig. 6.9. With use of (6.21) we calculate for x_{rms}

$$x_{\text{rms}} = \left[\frac{D^2 \omega}{2\pi} \int_{t_0}^{t_0 + \frac{2\pi}{\omega}} \{\sin(\omega t)\}^2 dt \right]^{\frac{1}{2}} = \sqrt{\frac{D^2 \omega t}{2\pi} \Big|_{t_0}^{t_0 + \frac{2\pi}{\omega}}} = \frac{1}{2} D \sqrt{2}.$$

The auto-correlation of the signal $x(t)$ becomes

$$R_{xx}(\tau) = \frac{\omega D^2}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin \omega t \sin \omega(t + \tau) dt = \frac{D^2}{2} \cos \omega \tau,$$

and

$$R_{xx}(0) = x_{\text{rms}}^2 = \frac{D^2}{2}.$$

End of Example

With the aid of the Parseval's theorem the average power of $x(t)$ can be expressed in the frequency domain [Papoulis 1962]

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \lim_{T \rightarrow \infty} \frac{1}{2\pi T} \int_{-\frac{T}{2}}^{\frac{T}{2}} X(\omega)X^*(\omega)d\omega = \lim_{T \rightarrow \infty} \frac{1}{2\pi} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{|X(\omega)|^2}{T} d\omega, \quad (6.25)$$

where $|X(\omega)|^2$ is the power spectrum, with

$$|z|^2 = zz^* = (x + jy)(x - jy) = x^2 + y^2, \quad X^*(\omega) = \int_{-\infty}^{\infty} x(t)e^{j\omega t} dt \text{ and}$$

$\lim_{T \rightarrow \infty} \frac{|X(\omega)|^2}{T}$ the power spectral density (PSD function) in general denoted with

$$S_{xx}(\omega) = \lim_{T \rightarrow \infty} \frac{|X(\omega)|^2}{T} \geq 0. \text{ The dimension of the PSD function is unit}^2/\text{Rad.}$$

Equation (6.25) can be written as follows

$$x_{\text{rms}}^2 = R_{xx}(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) d\omega. \quad (6.26)$$

The power spectral density (PSD) function $S_{xx}(\omega)$ of the function $x(t)$ is defined as the Fourier transform of its auto correlation function $R_{xx}(\tau)$, thus

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau, \quad (6.27)$$

and

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega\tau} d\omega. \quad (6.28)$$

The relations (6.27) and (6.28) form the Wiener–Kintchine relationship [Harris 1974].

The PSD function is symmetric with respect to $\omega = 0$, $S(\omega) = S(-\omega)$, thus

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_0^{\infty} 2S_{xx}(\omega) e^{j\omega\tau} d\omega = \frac{1}{2\pi} \int_0^{\infty} 2S_{xx}(\omega) \cos \omega\tau d\omega. \quad (6.29)$$

It is more practical to define the power spectral density in cycles per second (Hz, cps)

$$R_{xx}(\tau) = \int_0^{\infty} 2S_{xx}(\omega) \cos(2\pi f\tau) df = \int_0^{\infty} W_{xx}(f) \cos(2\pi f\tau) df, \quad (6.30)$$

where $W_{xx}(f) = 2S_{xx}(\omega)$ is the PSD function in the frequency domain (Hz, cps). The dimension of $W_{xx}(f)$ is unit^2/Hz (e.g. Pa^2/Hz , g^2/Hz , etc.).

The square root of the mean value $x(t)$ then is, see (6.26) and (6.30):

$$x_{\text{rms}} = \sqrt{R_{xx}(0)} = \sqrt{\int_0^{\infty} W_{xx}(f) df} \quad (6.31)$$

Example

The PSD $W(f)$ of a signal is a band-limited white noise with a constant value W_o in the frequency band $[f_1, f_2]$, as shown in Fig. 6.10.

Calculate the auto correlation function and the rms value of the signal.

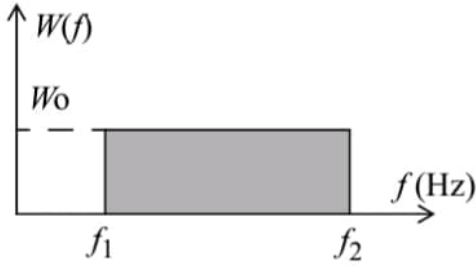


Fig. 6.10 Band limited white noise

The auto correlation function $R(\tau)$ is defined by (6.30)

$$R(\tau) = \int_0^{\infty} W(f) \cos(2\pi f\tau) df = W_0 \int_{f_1}^{f_2} \cos(2\pi f\tau) df,$$

$$R(\tau) = \frac{W_0}{2\pi\tau} [\sin(2\pi f_2\tau) - \sin(2\pi f_1\tau)].$$

Using the well-known limit $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ the rms value of the signal becomes

$$\text{rms} = \sqrt{R(0)} = \sqrt{W_0[f_2 - f_1]}.$$

The rms value of the signal can also be calculated with (6.31), as follows

$$\text{rms} = \sqrt{\int_0^{\infty} W_{xx}(f) df} = \sqrt{W_0[f_2 - f_1]}.$$

End of example

The definition of the unit decibel per octave (dB/oct) is given by

$$10 \log \left[\frac{W(f_2 = 2f)}{W(f_1 = f)} \right] = r \quad (\text{dB/oct}). \quad (6.32)$$

An octave band is given by

$$\frac{f_2}{f_1} = 2^1. \quad (6.33)$$

One octave is in fact the doubling of the frequency.

When the ratio between the frequency is not exactly a factor of 2 but slightly more or less, then the number of octaves is calculated in the following way:

$$\frac{f_y}{f_{\text{ref}}} = 2^y, \quad (6.34)$$

with f_y the considered frequency (Hz), f_{ref} the reference frequency (Hz) and y the number of octaves (Oct). Then y can be obtained as follows

$$y = \frac{\log\left(\frac{f_y}{f_{ref}}\right)}{\log 2} \approx 3.322 \log\left(\frac{f_y}{f_{ref}}\right). \quad (6.35)$$

In the case where the number of octaves y and the number of decibels r per octave are known, one can easily calculate the increase or decrease of the power spectral density (see (6.32)):

$$10 \log \left[\frac{W(f_y)}{W(f_{ref})} \right] = yr \quad (\text{dB}). \quad (6.36)$$

Then one can calculate the PSD function $W(f_y)$

$$W(f_y) = W(f_{ref}) 10^{\frac{yr}{10}} = W(f_{ref}) 10^{\frac{r \log\left(\frac{f_y}{f_{ref}}\right)}{10 \log 2}}. \quad (6.37)$$

Elaborating (6.37) gives:

$$W(f_y) = W(f_{ref}) \left(\frac{f_y}{f_{ref}} \right)^{\frac{r}{3}}, \quad (6.38)$$

with $\log 2 \approx 0.30 \approx \frac{1}{3}$.

Example

As an example the power spectral density $W_{xx}(f=20)$ is calculated, when the PSD function at $f=150$ Hz is $W_{xx}(f=150) = 0.04 \text{ g}^2/\text{Hz}$. The slope is $r = 6$ dB/Oct. using (6.38)

$$W(f=20) = W(f=150) \left(\frac{20}{150} \right)^{\frac{6}{3}} = 0.04 \left(\frac{20}{150} \right)^2 = 0.0007 \text{ g}^2/\text{Hz}.$$

End of example

The root mean square (rms) value is representative value of a random power spectrum. An example spectrum is illustrated in Fig. 6.11.

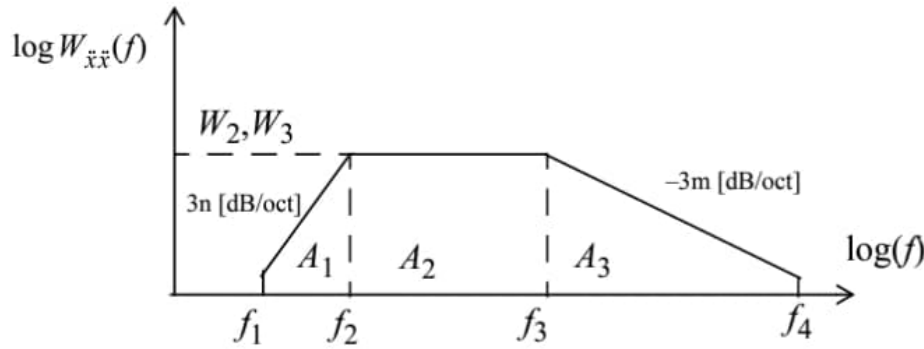


Fig. 6.11 PSD function $W_{\ddot{x}\ddot{x}}(f)$ versus frequency f (Hz) (very usual spectrum for Europe and USA)

The rms value of the acceleration \ddot{x}_{rms} is determined by:

$$\ddot{x}_{\text{rms}} = \sqrt{A_1 + A_2 + A_3} \quad (6.39)$$

where

$$A_1 = \frac{W_2 f_2}{n+1} \left[1 - \left(\frac{f_1}{f_2} \right)^{n+1} \right], \quad n > 0,$$

$$A_2 = W_2 (f_3 - f_2),$$

$$A_3 = \frac{W_3 f_3}{m+1} \left[\left(\frac{f_4}{f_3} \right)^{m+1} - 1 \right], \quad m < 0, \quad (m \neq -1).$$

For $m = -1$ (with the help of the rule of l'Hôpital):

$$A_3 = W_3 f_3 \ln \left(\frac{f_4}{f_3} \right) = 2.303 W_3 f_3 \log \left(\frac{f_4}{f_3} \right),$$

where W_2, W_3 are the power spectral densities (g^2/Hz) at the frequencies f_2 and f_3 (Hz) respectively, r_{12} is the slope (dB/Oct) between the frequencies f_1 and f_2 (Hz), and r_{34} is the slope (dB/Oct) between the frequencies f_3, f_4 (Hz),

$$n = \frac{\log \frac{W_2}{W_1}}{\log \frac{f_2}{f_1}} = \frac{\log \left(\frac{f_2}{f_1} \right)^{\frac{r_{12}}{3}}}{\log \frac{f_2}{f_1}} = \frac{r_{12}}{3} \quad \text{and} \quad m = \frac{\log \frac{W_4}{W_3}}{\log \frac{f_4}{f_3}} = \frac{\log \left(\frac{f_4}{f_3} \right)^{\frac{r_{34}}{3}}}{\log \frac{f_4}{f_3}} = \frac{r_{34}}{3}.$$

Example

Random accelerations at the base of the spacecraft for the ARIANE 4 are specified. The rms value for the acceleration spectrum below will be calculated (Table 6.6).

Table 6.6 Calculation of rms value

Frequency range (Hz)	Power Spectral Density (g^2/Hz)	Slope (dB/oct)	Area's (g^2)
20–150	+6dB/octave	$n = 2$	$A_1 = 2.0$
150–700	0.04		$A_2 = 22.0$
700–2000	-3dB/octave	$m = -1$	$A_3 = 29.4$
rms			$\sqrt{A_1 + A_2 + A_3} = 7.3$

End of example

6.5 Acoustic loads

The noise of the launch vehicle engines, the separation of the airflow along the launch vehicle and the aerodynamic noise generate acoustic loads in a broad frequency spectrum from 20–10000 Hz.

The acoustic loads also result in high frequency random vibration. The noise level is at its peak during lift-off and transonic flight of the launch vehicle.

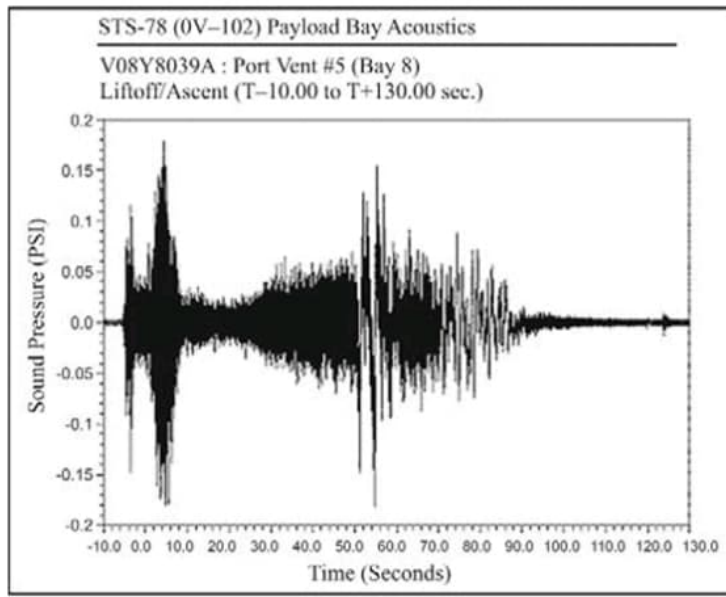


Fig. 6.12 High frequency sound pressures versus time (courtesy FEMCI, NASA GSFC)

An example of measured acoustic pressures is shown in Fig. 6.12 and an example of a specified acoustic load spectrum is illustrated in Fig. 6.13.

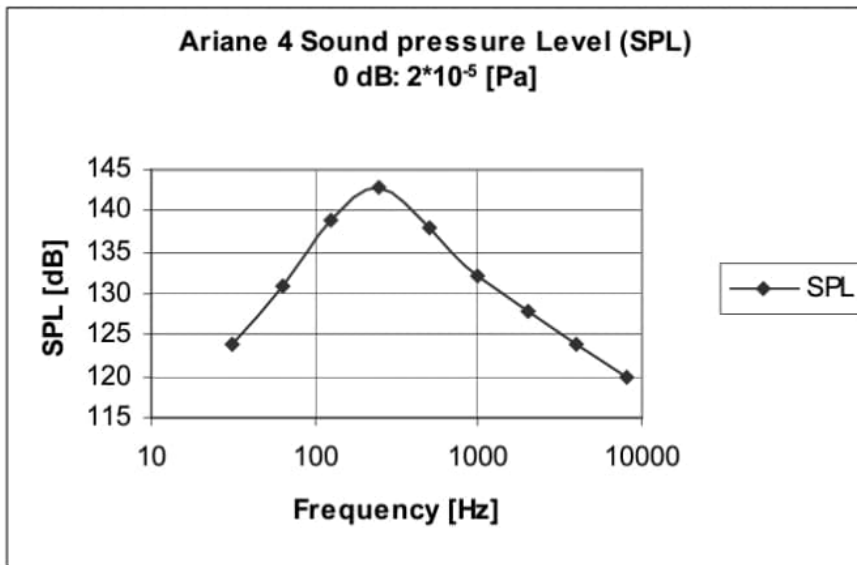


Fig. 6.13 Ariane 4 Acoustic loads, qualification level

The sound pressure level (SPL) is expressed in decibels (dB) and depends on the frequency. Various basic concepts will be explained in the following sections.

6.5.1 Sound Pressure Level

The sound pressure level (SPL) is generally given in decibels. The SPL gives an indication of the strength of the noise source but nothing about the direction.

In fact, a noise field is governed by two quantities: the sound pressure level and the direction. In a free space, a vibrating sphere will radiate sound in all directions, while in a closed space the noise field will reflect off the walls from several sides.

A noise field is called reverberant or diffuse when the noise strength is equally high from all directions. In the case of a reverberant noise field, the direction of sound is insignificant and only the noise strength is important.

The sound in a room consists of that coming directly from the source plus sound reflected or scattered by the walls and by objects in the room. Sound is called reverberant after having undergone one or more reflections [Pierce 1981].

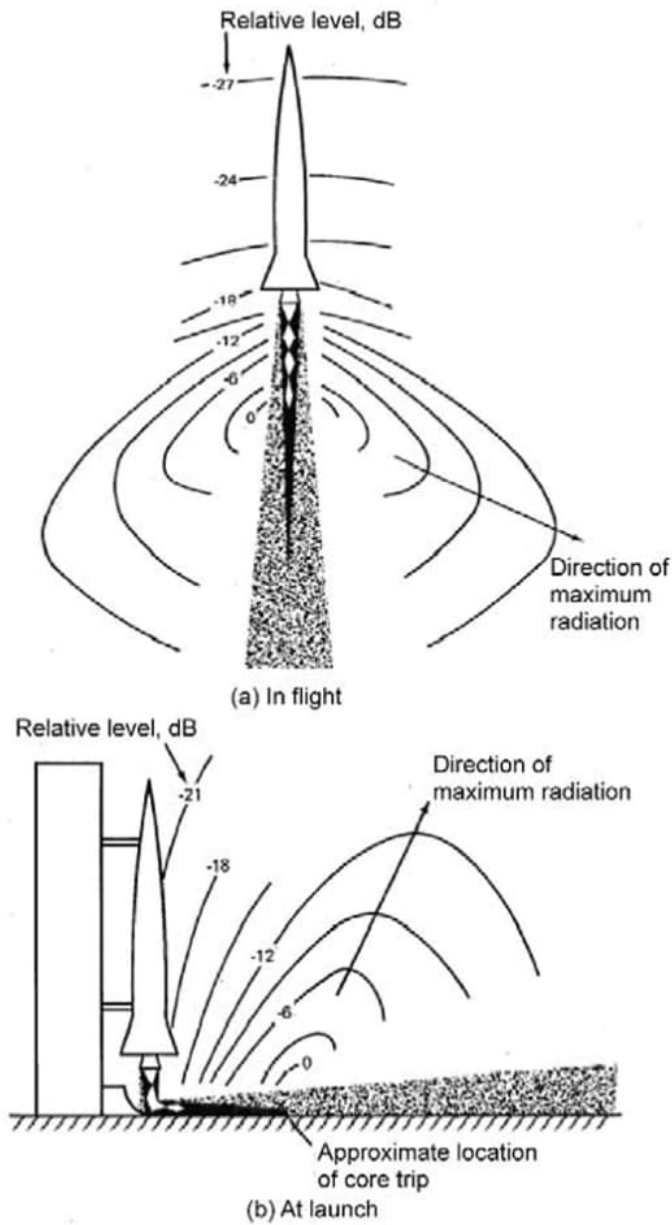


Fig. 6.14 Sketch of the rocket flow and contour overall sound-pressure level for flight and launch cases [NASA SP-8072]

Relative noise levels around a launch vehicle during lift-off and flight are shown in Fig. 6.14.

Examples of sound pressure levels are given in Table 6.7, [Pierce 1981].

Table 6.7 Examples

SPL (dB) $p_{\text{ref}} = 2 \times 10^{-5} \text{ Pa}$	Examples
140	Near jet engine (at 3 m)
130	Threshold of pain
120	Rock concert
110	Accelerating motorcycle (at 5 m)
100	Pneumatic hammer (at 2 m)
90	Noisy factory
80	Vacuum cleaner
70	Busy traffic

The exhaust noise of the engines causes considerable acoustic loads within the nose cone of the launch vehicle. The highest acoustic loads occur during lift-off and in transonic flight. Generally, a reverberant noise field is assumed. The strength of the noise field (SPL) is expressed in (dB), depending on the frequency. The frequency band is the octave- or one-third octave band. An example of a definition of acoustic loads is given in the following Table 6.8:

Table 6.8 Sound pressure level

Octave band	Sound Pressure Level (SPL) (dB) ref.: 0 (dB) = $2 \times 10^{-5} \text{ (Pa)}$
31.5	124
63	130
125	135
250	139
500	134
1000	128
2000	124
4000	120
8000	116
Overall Sound Pressure Level (OASPL)	142

The sound pressure level (SPL) is defined in the following way:

$$SPL = 10 \log \left(\frac{p^2}{p_{\text{ref}}^2} \right), \quad (6.40)$$

where the reference value of the sound pressure, $p_{\text{ref}} = 2 \times 10^{-5}$ Pa and p is the effective value of the occurring sound pressure.

The sound pressure is measured in a certain centre frequency with associated bandwidth.

In acoustics it is common to work with a constant relative bandwidth (the so-called octave or one-third octave band filters).

6.5.2 Octave band

For a constant relative bandwidth, the ratio between two consecutive frequencies is defined as:

$$\frac{f_x}{f_{\text{ref}}} = 2^x. \quad (6.41)$$

In which case it yields for x :

- $x = 1$ one speaks of an octave band, $\frac{f_x}{f_{\text{ref}}} = 2^1$ and when
- $x = \frac{1}{3}$ one speaks of a one-third octave band, $\frac{f_x}{f_{\text{ref}}} = 2^{\frac{1}{3}} = 1.260$

The centre frequencies in an octave- and one-third octave band are given in Table 6.9

6.5.3 Centre frequency

The centre frequency f_{cent} is the geometric mean of the minimum frequency f_{min} and the maximum frequency f_{max} in the relative frequency band, and is of course dependent on the octave band used. The centre frequency is:

$$f_{\text{cent}} = \sqrt{f_{\text{min}} f_{\text{max}}}. \quad (6.42)$$

6.5.4 Relative bandwidth

The bandwidth Δf is the difference between the maximum frequency f_{max} and the minimum frequency f_{min} and is given by:

$$\Delta f = f_{\max} - f_{\min} \cdot \quad (6.43)$$

The ratio between the extreme frequencies in the band is $\frac{f_{\max}}{f_{\min}} = 2^x$. It is then easy to derive the expression for the bandwidth in terms of the centre frequency:

$$\Delta f = \left(2^{\frac{x}{2}} - 2^{-\frac{x}{2}} \right) f_{\text{cent}} \cdot \quad (6.44)$$

Any proportional frequency band is defined by its centre frequency and by x . An octave band ($x = 1$) with a centre frequency 1000 Hz, the extreme frequencies of the frequency band are $f_{\min} = 707$ Hz and $f_{\max} = 1414$ Hz respectively and the relative bandwidth is $\Delta f = 707$ Hz.

Table 6.9 Centre frequencies octave and one-third octave frequency bands

Octave frequency band (Hz)	One-third octave frequency band (Hz)	Octave frequency band (Hz)	One-third octave frequency band (Hz)
31.5	25	1000	800
	31.5		1000
	40		1250
63	50	2000	1600
	63		2000
	80		2500
125	100	4000	3150
	125		4000
	160		5000
250	200	8000	6300
	250		8000
	315		10000
500	400		
	500		
	630		

The relative bandwidth for the one-octave and one-third octave bands are given in Table 6.10.

Table 6.10 Relative bandwidth

xst-Octave band	Bandwidth (Hz)
$x = 1$	$\Delta f = 0.7071f_{\text{cent}}$
$x = \frac{1}{3}$	$\Delta f = 0.2316f_{\text{cent}}$

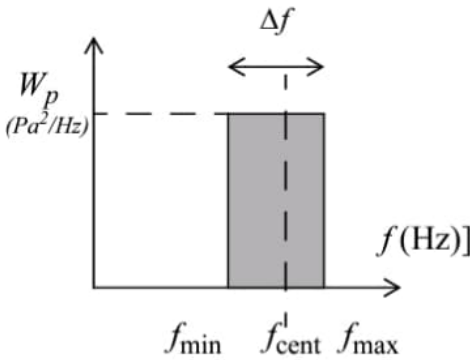
6.5.5 Power Spectral Density

The power spectral density of the effective (rms) sound pressure for a certain centre frequency with relative bandwidth Δf , is calculated as follows:

$$W_p(f_{\text{cent}}) = \frac{p^2}{\Delta f}, \quad (6.45)$$

in which $W_p(f_{\text{cent}})$ is the power spectral density of the sound pressure (Pa^2/Hz) and p^2 is the effective sound pressure.

The SPL is constant in the frequency bandwidth, hence the pressure is constant in the frequency bandwidth. That means that the power spectral density of the sound pressure is constant in the frequency bandwidth. This is illustrated in Fig. 6.15.

**Fig. 6.15** Calculation power spectral density

The square root of the mean value of the noise strength over the entire frequency band is calculated with (see (6.31)):

$$p_{\text{rms}} = \sqrt{\int_{f_{\text{lower}}}^{f_{\text{upper}}} W_p(f) df}. \quad (6.46)$$

Equation (6.46) can be simplified using (6.45):

$$p_{\text{rms}} = \sqrt{\int_{f_{\text{lower}}}^{f_{\text{upper}}} W_p(f) df} = \sqrt{\sum_k \frac{p_k^2}{\Delta f_k} \Delta f_k} = \sqrt{\sum_k p_k^2}. \quad (6.47)$$

The effective pressure p can be calculated with (6.40):

$$p_k^2 = p_{\text{ref}}^2 10^{\frac{SPL_k}{10}}. \quad (6.48)$$

The reference pressure is $p_{\text{ref}} = 2.0 \times 10^{-5}$ Pa, thus p_k can be written as follows:

$$p_k^2 = 10^{\frac{SPL_k - 94}{10}}. \quad (6.49)$$

The overall sound pressure level (OASPL) is calculated as follows:

$$OASPL = 10 \log \left(\frac{p_{\text{rms}}^2}{p_{\text{ref}}^2} \right). \quad (6.50)$$

This is in accordance with (6.40).

We may rewrite (6.50)

$$OASPL = 10 \log p_{\text{rms}}^2 + 94. \quad (6.51)$$

An example of a calculation of the OASPL is given in the following Table 6.11.

6.5.6 Conversions of SPL

The following relation determines the conversion of the 1/3-octave band to the one-octave band:

$$SPL_{1\text{-octave}} = 10 \log \left\{ \sum_{k=1}^3 10^{\frac{SPL_{\frac{1}{3}\text{-octave}}}{10}} \right\}. \quad (6.52)$$

One octave frequency band contains three 1/3-octave bands (see Table 6.9). The mean pressure in the octave band is equal to the sum of the mean square pressure in the 1/3-octave band (see (6.47)).

Table 6.11 Calculation OASPL

Octave band	Sound Pressure Level (SPL) (dB)		Sound pressure p_k^2 (Pa ²)
	ref: 0 [dB] = 2×10^{-5} (Pa)		
31.5	124		1.005×10^3
63	130		4.000×10^3
125	135		1.265×10^4
250	139		3.177×10^4
500	134		1.005×10^4
1000	128		2.524×10^3
2000	124		1.005×10^3
4000	120		4.000×10^2
8000	116		1.592×10^2
		$p_{rms}^2 = \sum_k p_k^2$	6.356×10^4
Overall Sound Pressure level (OASPL)	$OASPL = 10 \log \left(\frac{p_{rms}^2}{p_{ref}^2} \right)$		142 [dB]
Overall Sound Pressure level (OASPL)	142		

$$p_{octave}^2 = \sum_{k=1}^3 p_{\frac{1}{3}\text{-octave},k}^2 = \sum_{k=1}^3 p_{ref}^2 10^{\frac{SPL_{\frac{1}{3}\text{-octave},k}}{10}} \quad (6.53)$$

Dividing both sides of (6.53) by p_{ref}^2 then

$$\frac{p_{octave}^2}{p_{ref}^2} = \sum_{k=1}^3 10^{\frac{SPL_{\frac{1}{3}\text{-octave},k}}{10}} \quad (6.54)$$

By taking the 10 logarithm (log) for both sides and multiplying with 10, (6.54) becomes

$$SPL_{octave} = 10 \log \left(\frac{P_{octave}^2}{P_{ref}^2} \right) = 10 \log \left(\sum_{k=1}^3 10^{\frac{SPL_{\frac{1}{3}-octave, k}}{10}} \right). \quad (6.55)$$

The following relation determines the conversion of the 1-octave band to the 1/3-octave band:

$$SPL_{\frac{1}{3}-octave} = SPL_{1-octave} + 10 \log \left\{ \frac{\Delta f_{\frac{1}{3}-octave}}{\Delta f_{1-octave}} \right\}. \quad (6.56)$$

Example

The conversion of the 1-octave band to the 1/3-octave band is given Table 6.12. An example conversion of the 1/3-octave band to the 1-octave band is illustrated in Table 6.13

Table 6.12 Example conversion calculation 1–1/3 octave band

Octave band (Hz)	$SPL_{1-octave}$ (dB)	$\Delta f_{1-octave}$ (Hz)	1/3-octave band (Hz)	$\Delta f_{\frac{1}{3}-octave}$ (Hz)	$SPL_{\frac{1}{3}-octave}$ (dB)
125	135	88.4	100	23.2	129.2
			125	28.9	130.1
			160	37.1	131.2

Table 6.13 Example conversion calculation 1/3-1 octave band ($p_{ref} = 2 \times 10^{-5}$ Pa)

1/3-octave band (Hz)	$SPL_{\frac{1}{3}-octave}$ (dB)	$\frac{SPL_{\frac{1}{3}-octave}}{10}$	Octave band (Hz)	$SPL_{1-octave}$ (dB)
100	129.2	12.92	125	135
125	130.1	13.01		
160	131.2	13.12		

End of example

6.5.7 Acoustic Fill Factor

Often, the acoustic environment for launch vehicles is representative for the unfilled or empty environment. It becomes necessary to account for the presence of the payload fill and its effects on the interior sound pressure level, [Hughes 1994].

The fill factor FF is given by the following expression [Hughes 1994]

$$FF(f) = 10 \log \left[\frac{\left\{ 1 + \frac{c}{2fH} \right\}}{1 + \left\{ \frac{c}{2fH} (1 - V_{ratio}) \right\}} \right] \text{ (dB)} \quad (6.57)$$

where

- c is the speed of sound in air (m/s^2)
- f is the **one third octave band center frequency** (Hz)
- H is the gap distance between the payload and the fairing/cargo bay wall (m)
- V_{ratio} is the volume ratio of the payload volume to the empty fairing/cargo bay volume, for a given payload zone length.

Add the fill factor effect to the acoustic levels specified for the empty fairing/cargo bay. If the sound pressure levels are specified in the octave band a conversion to the one third octave band is needed to add the fill factor. After that, the octave band specification may be converted to the octave band.

Example

Let the factor $\frac{c}{fH} = 100$ and the volume ratio $V_{ratio} = 0.70$ then the fill factor $FF = 5$ dB. The empty volume $SPL(f) = 130$ dB. The total filled sound pressure level at the one third octave band center frequency f (Hz) becomes $SPL(f) = 135$ dB.

End of example

6.6 Shock loads

6.6.1 Introduction

Separation of stages and the separation of the spacecraft from the last stage of the launch vehicle will induce very short duration loads in the internal structure of the spacecraft, these are the shock loads. The duration of the shock load is in general very short with respect to the duration associated with the fundamental natural frequencies of the loaded dynamic mechanical system.

The effects of the shock loads are generally represented in a Shock Response Spectrum (SRS). The SRS is essentially a plot that shows the responses of a number of single degree of freedom (SDOF) systems to an excitation.

The excitation is usually an acceleration time history. This process is illustrated in Fig. 6.16.

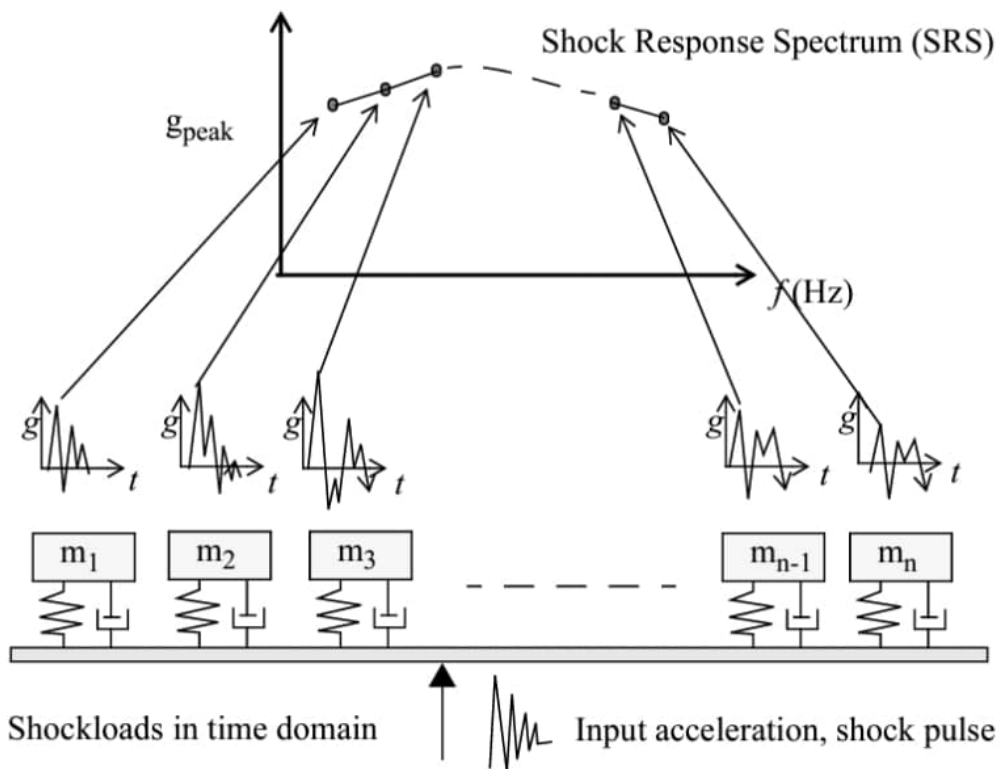


Fig. 6.16 The SRS (Shock Response Spectrum) concept. An input transient acceleration to be analysed is processed mathematically in a way that simulates the process represented here.

The spacecraft is generally loaded by the heaviest loads when the nose cone is fired away and when the spacecraft separates from the last stage of the launch vehicle. The combustion and the burn-up of the engines generally result in lower shock loads.

The launch authorities specify the shock load with a “shock spectrum”. An example of an ARIANE 4 shock spectrum is given in Fig. 6.17. The damping factor (Quality factor Q) must be specified.

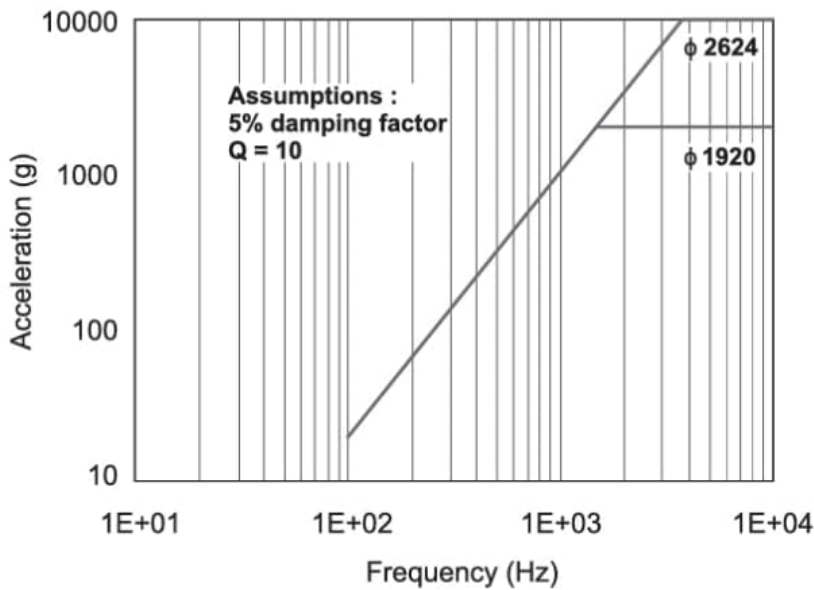


Fig. 6.17 Ariane 4 Shock Response Spectrum (SRS)

A SRS is generated by calculating the maximum response of a SDOF system to a particular base transient excitation. Many SDOF systems tuned to a range of natural frequencies are assessed using the same input time history. A damping value must be selected in the analysis. A damping ratio of $\zeta = 0.05$, $Q = 10$, is commonly used.

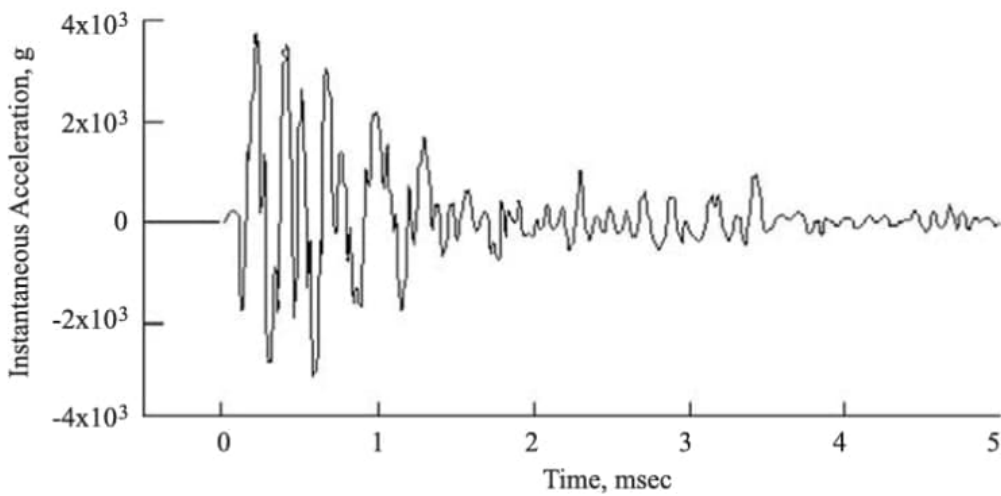


Fig. 6.18 Typical Pyroshock acceleration Time History [NASA-STD-7003]

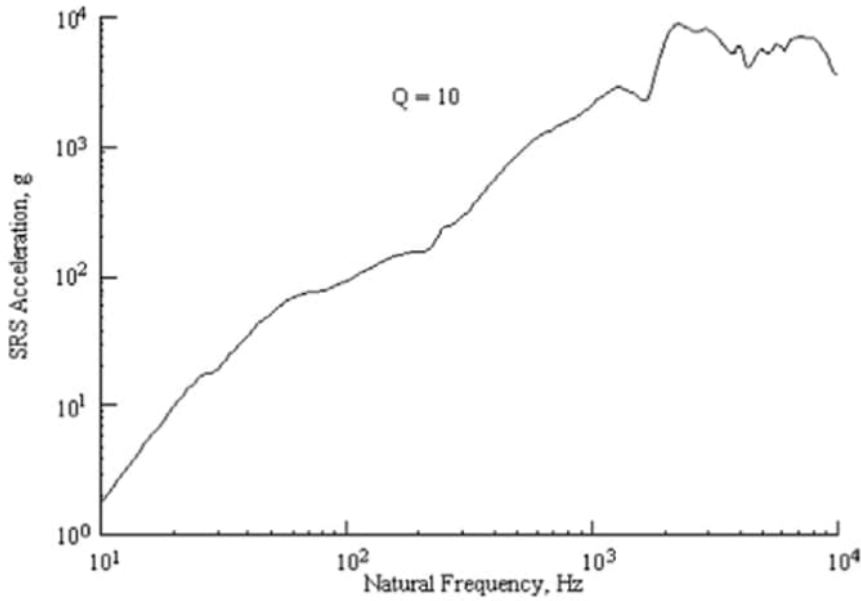


Fig. 6.19 Typical Pyroschock Maximum Shock Response Spectrum (SRS) [NASA-STD-7003]

The final SRS plot looks like a frequency domain plot. It shows the largest absolute response encountered for a particular SDOF system anywhere within the analysed time. Therefore the SRS provides an estimate of the response of an actual product and its various components to a given transient input (i.e. shock pulse). A typical example of a time history acceleration and associated SRS as illustrated in Fig. 6.18 and Fig. 6.19, are extracted from NASA-STD-7003.

In this section the response of a SDOF system, due to enforced acceleration, will be recapitulated.

6.6.2 Enforced acceleration

A SDOF system with a discrete mass m , a damper element c and a spring element k is placed on a moving base which is accelerated with an acceleration $\ddot{u}(t)$. The resulting displacement of the mass is $x(t)$. We introduce the natural (circular) frequency $\omega_n = \sqrt{\frac{k}{m}}$, the damped circular frequency $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, the critical

damping constant $c_{\text{crit}} = 2\sqrt{km}$ and the damping ratio $\zeta = \frac{c}{c_{\text{crit}}}$. The amplifica-

tion factor is defined as $Q = \frac{1}{2\zeta}$ where $Q = 10$ is generally assumed.

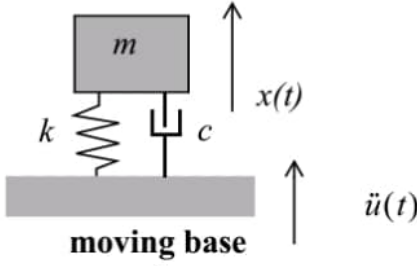


Fig. 6.20 Enforced acceleration on a damped SDOF system

We introduce a relative motion $z(t)$, which is the displacement of the mass with respect to the base. The relative displacement is

$$z(t) = x(t) - u(t). \quad (6.58)$$

The equation of motion for the relative motion $z(t)$ is

$$\ddot{z}(t) + 2\zeta\omega_n\dot{z}(t) + \omega_n^2z(t) = -\ddot{u}(t). \quad (6.59)$$

The enforced acceleration of the SDOF system is transformed into an external force. The absolute displacement $x(t)$ can be calculated with

$$\ddot{x}(t) = \ddot{z}(t) + \ddot{u}(t) = -2\zeta\omega_n\dot{z}(t) - \omega_n^2z(t). \quad (6.60)$$

The solution of (6.59), using initial conditions with respect to displacement $z(0)$ and velocity $\dot{z}(0)$ is

$$\begin{aligned} z(t) = & z(0)e^{-\zeta\omega_n t} \left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin\omega_d t \right) + \dots \\ & \dots + \dot{z}(0)e^{-\zeta\omega_n t} \frac{\sin\omega_d t}{\omega_d} - \int_0^t e^{-\zeta\omega_n \tau} \frac{\sin\omega_d \tau}{\omega_d} \ddot{u}(t-\tau) d\tau. \end{aligned} \quad (6.61)$$

For SRS calculations $z(0) = \dot{z}(0) = 0$, hence

$$z(t) = - \int_0^t e^{-\zeta\omega_n \tau} \frac{\sin\omega_d \tau}{\omega_d} \ddot{u}(t-\tau) d\tau = - \int_0^t e^{-\zeta\omega_n(t-\tau)} \frac{\sin\omega_d(t-\tau)}{\omega_d} \ddot{u}(\tau) d\tau. \quad (6.62)$$

After differentiation of (6.62) with respect to time, the relative velocity $\dot{z}(t)$ becomes

$$\dot{z}(t) = - \int_0^t e^{-\zeta\omega_n(t-\tau)} \cos(\omega_d(t-\tau)) \dot{\ddot{u}}(\tau) d\tau - \zeta\omega_n z(t). \quad (6.63)$$

The absolute acceleration $\ddot{x}(t)$ can be obtained applying (6.60)

$$\ddot{x}(t) = 2\zeta\omega_n \int_0^t e^{-\zeta\omega_n(t-\tau)} \cos(\omega_d(t-\tau)) \ddot{u}(\tau) d\tau + \omega_n(2\zeta^2 - 1)z(t). \quad (6.64)$$

The maximum acceleration $\ddot{x}(t)$ can be calculated by inserting the natural frequency $\omega_n = 2\pi f_n$ (Rad/s) of the SDOF system for every natural frequency. The maximum acceleration $\ddot{x}(t)$ will be plotted against the number of cycles per second f_n (Hz). This plot is called the Shock Response Spectrum (SRS) of the base excitation $\ddot{u}(t)$.

For the calculation of the SRS the following parameters are important:

1. The damping ratio ζ of the SDOF dynamic system.
2. The number of SDOF systems for which the maximum response is calculated
3. The minimum time frame of the transient period T_{\min} [s]. The minimum time frame is the larger of either $T_{\min} \geq \frac{1}{f_{\min}}$ or twice the maximum shock time $T_{\min} \geq 2t_{shock}$.
4. The time increment Δt must be less than 10% of the reciprocal value of the maximum frequency f_{\max} (Hz) involved in the calculation of the SRS, i.e. $\Delta t \leq \frac{0.1}{f_{\max}}$. The minimum number of time steps n within the time frame T_{\min} is $n = \frac{T_{\min}}{\Delta t} = 10 \frac{f_{\max}}{f_{\min}}$.

Example

A half sine pulse $\ddot{u}_{\text{base}} = 200 \sin \frac{\pi t}{\tau}$, $0 \leq t \leq \tau = 0.0005$ (s) and

$\ddot{u}_{\text{base}} = 0$, $t < 0$, $t > \tau$ is applied to the base of series of SDOF dynamic systems to calculate the SRS of the HSP. The total time is $t_{\text{end}} = 0.05$ s and

$$\Delta t = 0.00001 \leq \frac{0.1}{f_{\max}} = \frac{0.1}{3000} = 0.00003 \text{ s. The damping ratio } \zeta = 0.05,$$

$$Q = 10.$$

The calculated SRS (absolute acceleration) is illustrated in Fig. 6.21.

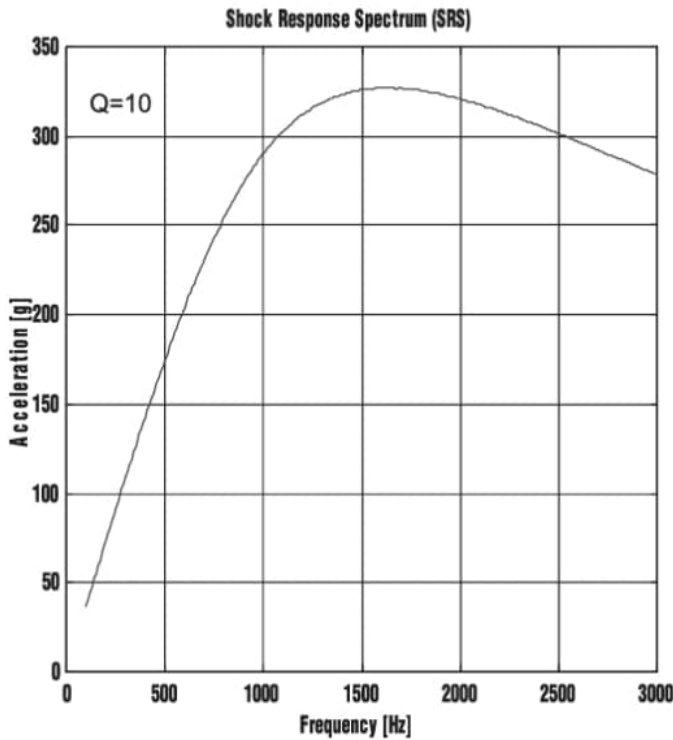


Fig. 6.21 The SRS of a Half Sine pulse (HSP) with amplitude $A=200\text{g}$ and a time duration $\tau = 0.0005\text{ s}$

6.6.3 Shock Attenuation Rules

A number of empirically derived shock attenuation rules have been proposed over the years by NASA and ESA. It is important to note that this assessment of attenuation is only valid for prediction of the shock environment induced by clampband separation.

The following scaling relationships are proposed [Kryenko 2004] by NASA and ESA respectively

$$att_{\text{NASA}} = e^{\left[\left(-8 \times 10^{-4} f^{\left\{ 2.4 f^{-0.105} \right\}} \right) d \right]}, \quad (6.65)$$

and

$$att_{\text{ESA}} = e^{\left[\left(-8 \times 10^{-4} f^{\left\{ 2.515 f^{-0.115} \right\}} \right) (0.0144 d^3 - 0.2 d^2 + 0.93 d + 0.024) \right]}, \quad (6.66)$$

where d (m) is the distance between the point of interest and the shock source and f is the frequency (Hz). These rules must be used with great care.

6.6.4 SRS Tolerance Limit

The SRS tolerance limit Tol_{SRS} is specified in (dB) and defined as follows

$$Tol_{SRS} = 20 \log \left\{ \frac{SRS_{Tol}}{SRS_{Nominal}} \right\} , \tag{6.67}$$

where SRS_{Tol} is the extreme value of the SRS defined by the tolerance band and $SRS_{Nominal}$ is the nominal specified value of the SRS. A very usual tolerance limit is $Tol_{SRS} = \pm 3$ dB.

6.7 Static pressure variations

During the launch phase, the pressure will decrease within the payload volume. Air cavities must be designed to have sufficient venting to prevent damage to the closing (surrounding) structure due to high pressure differences (pressure vessel).

The variation of the static pressure during launch is illustrated in Fig. 6.22.

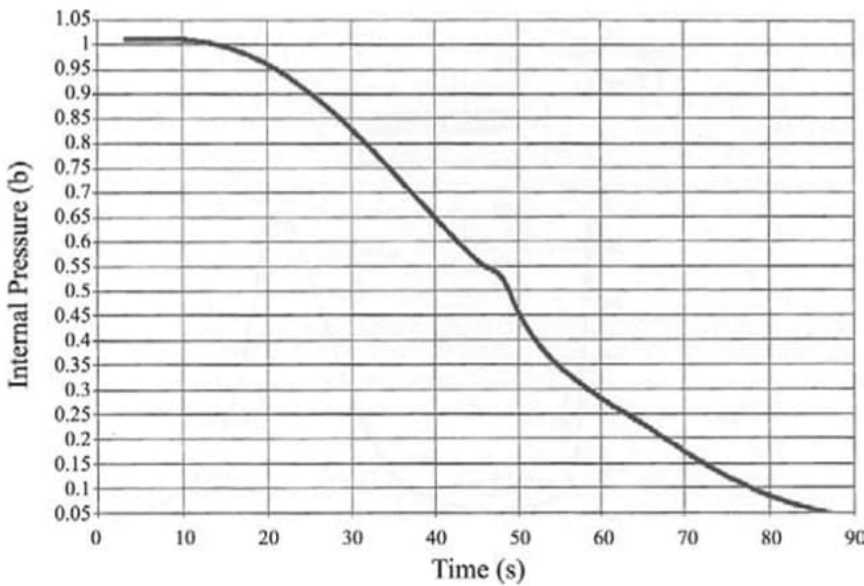


Fig. 6.22 Variation of static pressure within payload volume (Ariane 5)

As a rule of thumb sufficient venting will be provided if (ENVISAT-1 experience):

$$\frac{A}{V} \geq 20 \times 10^{-4} \text{ 1/m.} \quad (6.68)$$

where A is the total area of the venting holes (m^2) and V the total volume to be vented (m^3).

6.8 Micro-meteorites / Orbital Debris

6.8.1 Introduction

Space surrounding Earth is full of millions of micro meteoroids and man-made orbital debris. In the last 30 years that humans have been exploring space, much orbital debris has been created that poses a serious threat to spacecraft orbiting the earth. Orbital debris consists not only of large redundant stages of rockets and old spacecraft but, also small parts such as bits of paint and other fragments. Even minute parts can seriously damage a spacecraft because these parts move at very high velocities. Orbital debris flies with a velocity of 7.5 km/s (27000 km/h) in an orbit around the earth. If two parts have a frontal collision the crash velocity is therefore 15 km/s.

Micro meteorites usually circle the sun with speeds that can exceed 70 km/s. However, they are much smaller and their density is less than that of orbital debris.

Orbital debris is still increasing. Every year more and more spacecraft are launched into space, which then results in even more orbital debris. "Dead" spacecraft explode or disintegrate which results in thousands of new pieces of orbital debris.

Large parts can be traced by radar so that one can map the positions of orbital debris. The small parts that cannot be observed by radar however, are, nevertheless, dangerous for spacecraft.

Spacecraft must be designed against the impact of small parts at extremely high velocities.

In an orbit around the Earth, the parts (solar panels, antennas, radiators, etc.), boxes, and instruments mounted on the outside of the spacecraft are exposed to micro meteorites and man-made debris. In some cases protective measures must be taken.

A meteorite flux model describes the number of micro meteorites. The flux F of the micrometric is given as a function of the "particle" mass m (gram).

A debris flux model describes the debris. The flux F of the debris is given as a function of the "particle" diameter D (cm). The flux F describes the number of particles per m^2 , per year.

The density of the meteorites is $\rho = 0.5 \text{ gram/cm}^3$ for all sorts of meteorites and all existing sizes. The density of the debris with a diameter $D \leq 1 \text{ cm}$ is $\rho = 2.8 \text{ gram/cm}^3$. The density of the debris decreases as the diameter D increases.

A more detailed discussion about micro-meteoroids and space debris is given chapter 24 "Damage to Spacecraft by Meteoroids and Orbital Debris", page 399.

6.8.2 Simple Micro Meteoroid Flux Model

The micro meteoroid flux model is limited by a "particle" mass $10^{-12} \leq m \leq 1 \text{ gram}$.

For a "particle" mass $10^{-12} \leq m \leq 10^{-6} \text{ grams}$ the following micro meteorites flux model may be used:

$$\log F = -0.063(\log m)^2 - 1.584 \log m - 14.339 \text{ (Particles/m}^2\text{/year)}, \quad (6.69)$$

and for a "particle" mass $10^{-6} \leq m \leq 1 \text{ gram}$ the following micro meteorites flux model may be used:

$$\log F = -1.213 \log m - 14.37 \text{ (Particles/m}^2\text{/year)}, \quad (6.70)$$

where F is the average number of particles, with mass m or larger, per m^2 of surface area and per second and m the mass of the "particle" in grams.

6.8.3 Simple Debris flux Model

Orbital debris is defined as any man-made object in orbit about the Earth which no longer serves a useful purpose.

The higher the altitude, the longer the orbital debris will generally remain in Earth orbit. Debris left in orbits below 600 km normally fall back to Earth within a few years. At altitudes of 800 km, the time for orbital decay is often measured in decades. Above 1000 km, orbital debris will normally continue circling the Earth for a century or more.

The debris flux model is limited by a "particle" diameter $D \leq 2 \text{ (cm)}$.

For a "particle" $D \leq 1 \text{ (cm)}$ the following debris flux model may be used:

$$\log F = -2.52 \log D - 5.46 \text{ (Particles/m}^2\text{/year)}. \quad (6.71)$$

For a “particle” diameter $1 \leq D \leq 2$ cm the following debris flux model may be used:

$$\log F = -1.395 \log D - 5.46 \text{ (Particles/m}^2\text{/year)}, \quad (6.72)$$

where F is the average number of particles, with diameter D (cm) or larger, per m^2 of surface area and per year and D the size of the “particle” in (cm).

Much more information about Orbital debris can be read in [NRC 1995].